TEMPERATURE DOUBLING OF RESONANCE FREQUENCY IN DICKE MODEL

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It is found that doubling of a resonance frequency in the Dicke model increases with decreasing temperature. If the atom density is greater than the threshold density one of the resonance frequencies tends to zero when the temperature decreases from high temperatures to the phase transition point (soft mode).

Investigations of the dynamic susceptibility of a system of two-level atoms interacting with a radiation field (Dicke model) show that in the spontaneous coherent state (SCS) the resonance frequency of system is essentially defined by the temperature [1, 2]. A thermodynamically ordered phase SCS appears for temperatures $T \leqslant T_c$ if the threshold condition $\lambda > \omega$ is satisfied when the interaction constant of atoms with the radiation field exceeds a frequency of this field [3, 4]. In this note it is shown that a temperature anomaly of resonance frequency doubling exists in the non-ordered phase too with $\lambda < \omega$ or $\lambda > \omega$ but $T > T_c$.

The Hamiltonian of the Dicke model in the single mode case may be written as [5] ($\hbar = 1$)

$$H = \sum_{\substack{i=1 \\ \nu=1,2}}^{N} (\epsilon_\nu - \mu) a_{i\nu}^{+} a_{i\nu} + \omega b^{+} b + \frac{\lambda}{\sqrt{N}} \sum_{i=1}^{N} (a_{i2}^{+} a_{i1} B e^{i\boldsymbol{q}\boldsymbol{r}_i} + \text{h.c.}), \tag{1}$$

where $\lambda = d\sqrt{2\pi\omega\rho}$, ρ the atom density, d a matrix element of the atom dipole transition and the resonance case $\epsilon_2 - \epsilon_1 = \omega$ is supposed. From the condition $\Sigma_{\nu=1,2} \langle a_{i\nu}^{+} a_{i\nu} \rangle = 1$ it follows that $\epsilon_\nu - \mu = \pm\omega_0/2$.

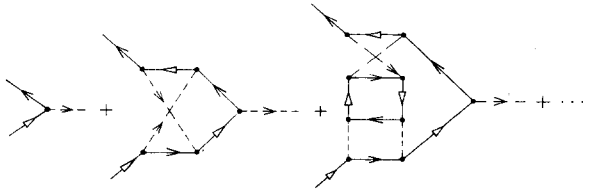
The transverse dynamic susceptibility of the system $\chi_{+-}(\Omega)$ is determined by analytical continuation of the temperature Green function [6]

$$\chi_{+-}(\omega_m) = (\Pi^{-1}(\omega_m) - (\lambda^2/N) F^0(\omega_m))^{-1} \tag{2}$$

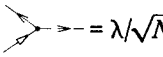
where Π is the polarization operator, $\omega_m = 2\pi m/\beta$. The zero-order photon Green function has the form

$$F^0(\omega_m) = \langle T(bb^{+}) \rangle_0 = -(i\omega_m - \omega)^{-1}.$$

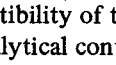
Let us consider some set of vertex diagrams



where



Ordinary and triangular arrows mean lower and upper states of the atom. All vertex diagrams become inessential in comparison with initial diagram $\lambda/\sqrt{N}$ in the thermodynamic limit $N \to \infty$, $\rho = \text{const}$. Besides, in this limit the self-energy operators for the fermion Green function become equal to zero when the atoms plus field system is in a thermodynamically non-ordered phase $T > T_c$. Note that this simplification of the problem does not take place when the atom subsystem is in open space.

As a result the polarization operator reduces to one loop  only, where according to (1) the zero-